Modelling a Superconducting Current Limiter using a Modified Broyden's Method

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Abstract — An efficient approach to solving a nonlinear algebraic equation set resulting from the application of the Finite Difference Method (FDM) is presented in the paper. The method is a modification of the Broyden's method, and is effective for modelling anisotropic and nonlinear electromagnetic devices and systems. The method is convergent in cases where other methods fail to converge. A case of modelling a superconducting short-circuit current limiter with an open magnetic core is used to illustrate the new method's application and effectiveness.

I. Introduction

Newton-Raphson (N-R) method and its modifications are fundamental methods for solving sets of nonlinear algebraic equations in electromagnetics. Its use requires resource-intensive computations of the Jacobi matrix of the system partial derivatives. The original Broyden's method [1] produces iteration updates of the Jacobian components without solving partial derivatives.

In most typical cases in the solution process of a set of nonlinear equations, the Jacobian in the first or in consecutive iterations is approximated by the Broyden's method e.g. to electromagnetic optimisation problems [2].

Extended Broyden's method is also used in optimisation procedures, e.g. by using a sensitivity analysis in the frequency domain [3] that relies on the system matrix. In addition to the typical use of the method to update the system derivatives, its robustness is improved by switching between the Broyden and the finite difference estimation of the system derivatives.

The newly proposed modification of the Broyden's method relies upon a so called *useful update u* that is used to calculate the next approximation of the magnetic vector potential. A solenoid with a magnetic core was used in [4] to illustrate the new method's application for a stationary magnetic field case.

In this paper a quasi-stationary magnetic field of a superconducting short-current limiter is modelled by the new method and its solution is compared with results of the simple iteration method and a simplified N-R method.

II. The method

In the Broyden's method [1] the update s_i for *i-th* iteration is calculated from:

$$D \cdot s_i = -f\left(A^{(i)}\right) \tag{1}$$

Where D is the matrix that approximates the Jacobian, f is the function describing the magnetic field in a point of a given discretisation mesh of the considered area, and A denotes the magnetic vector potential.

The next iteration value is determined from:

$$A^{(i+1)} = A^{(i)} + s_i \tag{2}$$

The next values of D are calculated from:

$$D_{i+1} = D_i + \frac{f\left(A^{(i+1)}\right)s_i^T}{s_i^T s_i} \tag{3}$$

In most cases of anisotropic and/or nonlinear electromagnetic configurations the Broyden's solution is either non- or very-slowly convergent.

We can now calculate the newly proposed *useful update* u from the following equation:

$$E\left(A^{(i)}\right) \cdot u_i = s_i \tag{4}$$

Where $E(A^{(i)})$ is the matrix of arguments calculated from the last approximation of $A^{(i)}$. The next approximation of $A^{(i)}$ is evaluated from:

$$A^{(i+1)} = A^{(i)} + u_i \tag{5}$$

The values of (5) allow us to calculate the values of reluctivity $\rho = \rho(A)$ needed to determine the matrix $E(A^{(i)})$ in the area of a ferromagnetic core.

III. The problem

Superconducting induction current limiters are used to limit short-circuit currents in power systems. The limiter to be analysed has a relatively simple construction with an open magnetic core (Fig. 1). In a normal superconducting operational state of the limiter, the magnetic field in the core is very small due to the 'screening' effect of the large current in the secondary superconducting winding. This is confirmed by the results of modelling.

The resistive state of the limiter occurs when the primary short-circuit current causes the loss of superconductivity of the secondary winding; its resistance and magnetic permeability (and magnetic flux density) of the core increase significantly. The input impedance of the limiter increases, effectively attenuating the short-circuit current in the power system.

There are four different material areas in the limiter: the surrounding air (1), the primary winding (2), the secondary winding – a superconducting ring (3) and the ferromagnetic core (4), characterised by their respective magnetic permeabilities.

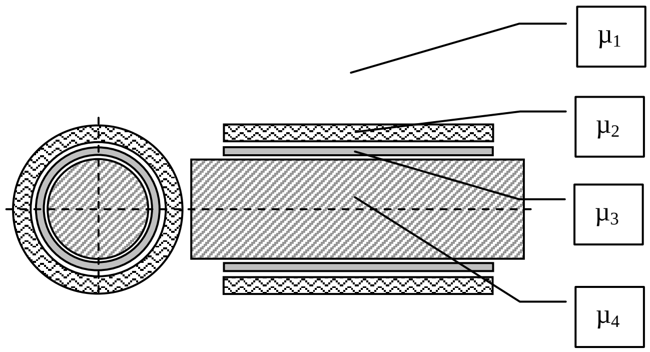

Fig. 1. The configuration of the current limiter

IV. The Solution

The fundamental equation for a quasi-stationary two dimensional magnetic field has the form:

$$rot\left(\frac{1}{\mu}\cdot rot\boldsymbol{A}\right) = -\boldsymbol{J} + \gamma\frac{\partial \boldsymbol{A}}{\partial t} \qquad (6)$$

A and *J* are complex numbers. Equations resulting from (6) for the four areas of the limiter and those for boundaries between the areas are converted to a finite difference form and solved by the proposed method after discretisation of the volume using a rectangular, non-uniform mesh. An example of a solution of the magnetic flux density in the limiter is shown in Fig. 1.

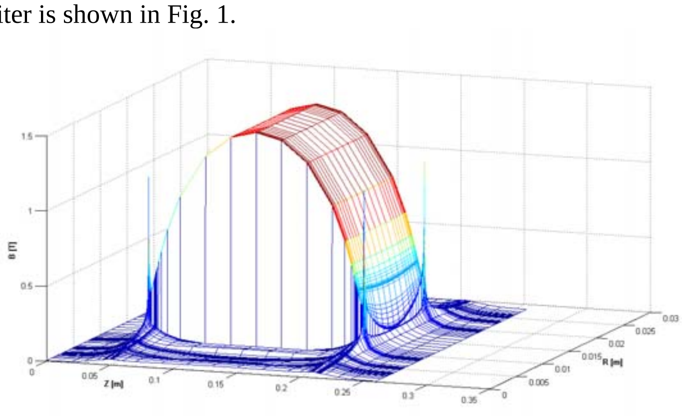

Fig. 2. Distribution of the magnetic flux density B (T) in the current limiter in the resistive state - view from the primary winding

V. Comparison of Results

Fig. 3 and Fig. 4 show the behavior of the Broyden's update (that fails to converge in the computational process) and the modified Broyden's update proposed by the authors that converges successfully.

The condition of the termination of iterations is checked using the square norm (the norm) defined below, where ε is the required iteration accuracy.

$$\sum_{i=1}^{w} f_i(x)^2 < \varepsilon \qquad (6)$$

Comparison of the norm for three computational solution methods for the current limiter: the modified Broyden's method, simplified Newton-Raphson (simplified Jacobian elements) and simple iteration methods is shown in Table 1.

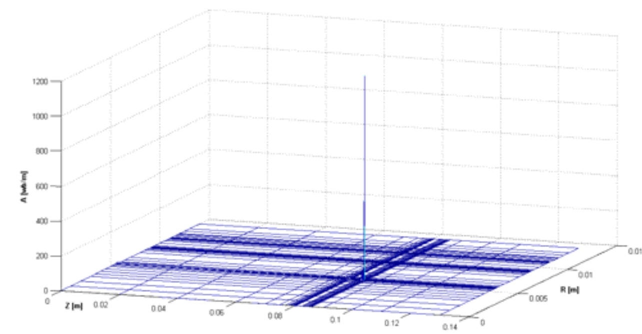

Fig. 3. Results of Broyden's update A (Wb/m)

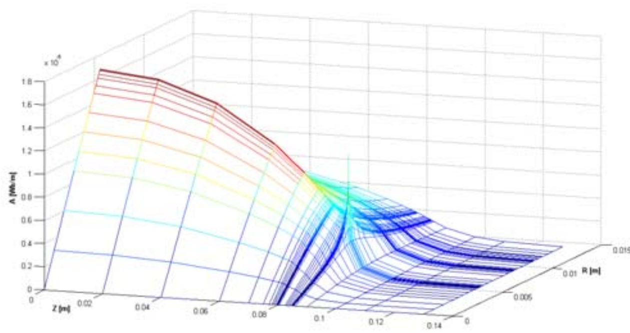

Fig. 4. Results of the modified Broyden's update A (Wb/m)

TABLE I
COMPARISON OF THE NORM FOR THREE METHODS

Method	**Value of the norm**	**Number of iterations**
The proposed Broyden's modification	$3.6\cdot10^{-4}$	19
Simplified N-R method	$4.8\cdot10^{12}$	25
The simple iteration method	$1\cdot10^{23}$	25

The proposed modification of the Broyden's method delivers accurate and accelerated results for an electromagnetic field solution in an object with nonlinearities and with material discontinuities as compared with a classical Broyden's, a simplified N-R and the simple iteration methods that may not converge at all.

VI. References

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